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Laser-dressing and magnetic-field effects on shallow-donor impurity states in semiconductor GaAs–$Ga_{1-x}Al_xAs$ cylindrical quantum-well wires

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Abstract
The influence of an intense laser field on shallow-donor states in cylindrical GaAs–$Ga_{1-x}Al_xAs$ quantum-well wires under an external magnetic field applied along the wire axis is theoretically studied. Numerical calculations are performed in the framework of the effective-mass approximation, and the impurity energies corresponding to the ground state and $2p_{\pm}$ excited states are obtained via a variational procedure. The laser-field effects on the shallow-donor states are considered within the extended dressed-atom approach, which allows one to treat the problem 'impurity + heterostructure + laser field + magnetic field' as a renormalized 'impurity + heterostructure + magnetic field' problem, in which the laser effects are taken into account through a renormalization of both the conduction-band effective mass and fundamental semiconductor gap.

(Some figures in this article are in colour only in the electronic version)

1. Introduction

The role played by impurity states in low-dimensional semiconductor systems has received great attention in the last few decades. From the experimental point of view, impurity states are responsible for a wide variety of important effects on phenomena such as conductivity, optical properties [1, 2], impurity-assisted tunneling [3], impurity breakdown [4], etc, taking place in semiconductors and its heterostructures. Impurity effects are also determinant in several transport properties which are useful in tailoring many technological aspects of the design and construction of electronic and optoelectronic devices. In that respect, a considerable number of studies has been devoted to the understanding of the basic properties of shallow donors in semiconductor heterostructures. For a number of reasons, particular interest has been paid to shallow-impurity properties in GaAs–$Ga_{1-x}Al_xAs$ quantum wells (QWs), quantum-well wires (QWWs), and quantum dots under the influence of external electric and magnetic fields, applied hydrostatic pressure, and so on [5–8]. Together with the intensive investigations on the shallow-impurity properties in semiconductor heterostructures, the understanding of the interaction mechanism between a laser field and semiconductor heterostructures has also attracted the attention of the research community for a long time. This interest has been motivated, in part, due to the possibility of designing new efficient optoelectronic devices [9, 10]. In the particular case of the interaction between an intense laser and a shallow impurity in semiconductor heterostructures, laser-field effects may be detected, for instance, by measuring the changes occurring both in the binding energy and in the internal transition energies associated with the impurity states, and which show up as modifications of the optical properties in the semiconductor heterostructure. Recently, it has been shown that modifications on the intra-donor transitions due to the presence of a laser acting upon a GaAs–$Ga_{1-x}Al_xAs$ QW may be as important as the effects of a strong magnetic field [11–13].

In the present work we study the properties of a shallow donor in a semiconductor GaAs–$Ga_{1-x}Al_xAs$ QWW by using a recent theoretical model resulting from an extension of the dressed-atom approach [14] and devoted to account for the interaction between a laser field and a semiconductor heterostructure [11–13]. In such a model, which may be termed as the extended dressed-atom approach, the effects of the laser field are incorporated via a renormalization of both the fundamental semiconductor energy gap and the conduction-band effective mass associated with the semiconductor heterostructure. Many-body interaction effects are not relevant for the present purposes, and may be taken as small corrections to the one-electron approximation, a physical situation which is essentially valid when the laser frequency is tuned far from any resonance. Together with the laser-field effects, we have also considered the influence of an external magnetic field applied along the wire axis on the ground state and $2p_{\pm}$ excited states of the donor located over the wire axis. This work is organized as follows. In section 2 we outline the extended dressed-atom approach and briefly discuss the quantum-mechanical problem of an on-center impurity electron in a cylindrical QWW. Section 3 presents our numerical results and discussion, and conclusions are given in section 4.

2. Theoretical framework

As is well known from the extended dressed-atom approach [11–13], the incidence of a monochromatic homogeneous laser field with frequency ω on a $Ga_{1-x}Al_xAs$ bulk semiconductor leads to a renormalization of the conduction-band effective mass m^* and energy gap E_g in such materials, i.e. the impurity-electron effective mass as well as the width of the fundamental gap become dependent on the laser detuning $\delta = E_g^0 - \hbar\omega$ and laser intensity I, where E_g^0 is the GaAs fundamental gap in the absence of laser radiation (see below). Of course, the effective masses and gaps are also dependent [15] on the Al fraction in bulk $Ga_{1-x}Al_xAs$, leading to an x-dependent renormalized electron effective mass and fundamental gap in the heterostructure.

2.1. The extended dressed-atom approach

The aim of the extended dressed-atom approach is to theoretically describe the effects of a homogeneous laser field on the semiconductor band structure. Such a description is carried out within the $\mathbf{k}\cdot\mathbf{p}$ procedure, in which the Kane model [2] is used to describe the conduction Γ_6^c and valence $\Gamma_{7,8}^v$ states. In order to derive the fundamental expressions of the dressed-atom approach we work within the diagonal representation of the Kane Hamiltonian, i.e. we use as a basis the states $|\Gamma_6^c\rangle$, $|\Gamma_8^v\rangle$ and $|\Gamma_7^v\rangle$ obtained from the diagonalization of the Kane matrix. The Hamiltonian describing the laser-field effects on the band structure is given by [12]

$$\hat{h} = \hat{h}_0 + \hbar\omega\hat{a}^\dagger\hat{a} + \frac{e}{m_0 c}A_\omega\hat{\mathbf{p}}\cdot\hat{\boldsymbol{\epsilon}}(\hat{a}^\dagger + \hat{a}), \tag{1}$$

where $\hat{h}_0$ is the diagonal representation of the Kane Hamiltonian, $\hat{a}$ and $\hat{a}^\dagger$ are the annihilation and creation operators, respectively, of a photon corresponding to a laser mode with frequency ω and polarization $\hat{\boldsymbol{\epsilon}}$, $\hat{\mathbf{p}}$ is the momentum operator, and $A_\omega = \sqrt{2\pi\hbar c^2/\omega\Omega}$ is the vacuum field amplitude in the volume Ω. The Hamiltonian (1) may be diagonalized within the $|\Gamma_6^c, N\rangle \equiv |\Gamma_6^c\rangle \otimes |N\rangle$, $|\Gamma_8^v, N+1\rangle \equiv |\Gamma_8^v\rangle \otimes |N+1\rangle$ and $|\Gamma_7^v, N+1\rangle \equiv |\Gamma_7^v\rangle \otimes |N+1\rangle$ manifold. One may fold back the resulting 3×3 matrix and obtain a 2×2 matrix which takes into account the laser coupling between the $|\Gamma_6^c, N\rangle$ and $|\Gamma_7^v, N+1\rangle$ states as a second-order energy correction. Here $|N\rangle$ denotes a Fock state with $N \approx N_0 \gg 1$, where N_0 is the average number of photons in the laser field. In this way we obtain [12]

$$\hat{h} = \begin{pmatrix} E(\Gamma_6^c) + \Delta E_{\Gamma_6^c} + \Delta E'_{\Gamma_6^c} + N\hbar\omega & \Sigma \\ \Sigma & E(\Gamma_8^v) + \Delta E'_{\Gamma_8^v} + (N+1)\hbar\omega \end{pmatrix}, \tag{2}$$

where

$$\Delta E_{\Gamma_6^c} \approx \frac{\Lambda_0^2}{3\delta'}\left[1 - \frac{m_0}{\mu_{67}}\frac{E_k}{\delta'} - \frac{8E_kE_p}{3}\left(\frac{1}{(E_g^0)^2} + \frac{2}{(E_g^{0\prime})^2} + \frac{2}{E_g^0E_g^{0\prime}}\right)\right], \tag{3}$$

$$\Delta E'_{\Gamma_6^c} \approx \frac{2}{3}\frac{\Lambda_0^2}{\Lambda_1}\left[1 - \frac{m_0}{\mu_{68}}\frac{E_k}{\Lambda_1} - \frac{4E_kE_p}{3}\left(\frac{8}{(E_g^0)^2} + \frac{1}{(E_g^{0\prime})^2} + \frac{2}{E_g^0E_g^{0\prime}}\right)\right], \tag{4}$$

and

$$\Sigma^2 \approx \frac{2\Lambda_0^2}{3}\left[1 - \frac{4E_kE_p}{3}\left(\frac{8}{(E_g^0)^2} + \frac{1}{(E_g^{0\prime})^2} + \frac{2}{E_g^0E_g^{0\prime}}\right)\right]. \tag{5}$$

In the above equations, $E_g^0 = E(\Gamma_6^c) - E(\Gamma_8^v)$ is the fundamental gap, $\Lambda_0 = \frac{eA_0|p|}{2m_0c}$, $A_0 = 2\sqrt{N_0}A_\omega$ is the classical amplitude of the photon vector potential, $E_k = \frac{\hbar^2k^2}{2m_0}$, $E_p = \frac{|p|^2}{2m_0}$, p is the interband matrix element describing the coupling between the s states of the Γ_6^c conduction band with the valence states corresponding to Γ_8^v and Γ_7^v, and m_0 is the free-electron mass. Moreover, $E_g^{0\prime} = E_g^0 + \Delta$, $\Delta = E(\Gamma_8^v) - E(\Gamma_7^v)$ is the split-off valence gap, $\delta' = \delta + \Delta$, $\Lambda_1 = E_g^0 + \hbar\omega$, and $\frac{1}{\mu_{67(8)}} = \frac{1}{m_{\Gamma_6^c}} - \frac{1}{m_{\Gamma_{7(8)}^v}}$, where $m_{\Gamma_6^c}$, $m_{\Gamma_7^v}$, and $m_{\Gamma_8^v}$ are the effective masses associated with the conduction electrons in Γ_6^c, heavy holes in Γ_8^v, and holes in Γ_7^v, respectively [2, 12].

By diagonalizing (2), one may straightforwardly obtain the renormalized conduction (+) and valence (−) bands as

$$E_\pm(k) = \frac{E_g^0 \pm \hbar\omega}{2} + \frac{\Lambda_0^2}{6\delta'} + \frac{\hbar^2k^2}{2m_\pm} \pm \frac{1}{2}\sqrt{\frac{8\Lambda_0^2}{3} + \left(\delta + \frac{\Lambda_0^2}{3\delta'} + \frac{4\Lambda_0^2}{3\Lambda_1}\right)^2}, \tag{6}$$

with the corresponding effective mass $m_\pm$ given by

$$\frac{1}{m_\pm} = \frac{1}{2M}\left[1 + \frac{M}{\mu_{68}}\left(\frac{\Lambda_0^2\beta_\gamma}{3\delta'} \pm \Pi\right)\right], \quad (7)$$

where $\frac{1}{M} = \frac{1}{m_{\Gamma_6^c}} + \frac{1}{m_{\Gamma_8^v}}$,

$$\Pi = \frac{\left(1 + \frac{\Lambda_0^2\beta_\gamma}{3\delta'} + \frac{4\Lambda_0^2\beta_{\gamma'}}{3\Lambda_1}\right)\left(\delta + \frac{\Lambda_0^2}{3\delta'} + \frac{4\Lambda_0^2}{3\Lambda_1}\right) + \frac{4\Lambda_0^2\beta_{\gamma''}}{3}}{\sqrt{\frac{8\Lambda_0^2}{3} + \left(\delta + \frac{\Lambda_0^2}{3\delta'} + \frac{4\Lambda_0^2}{3\Lambda_1}\right)^2}}, \quad (8)$$

$$\beta_\gamma = -\frac{\mu_{68}}{\mu_{67}}\frac{1}{\delta'} + \frac{8E_p}{3}\frac{\mu_{68}}{m_0}\left(\frac{1}{(E_g^0)^2} + \frac{2}{(E_g^{0\prime})^2} + \frac{2}{E_g^0E_g^{0\prime}}\right), \quad (9)$$

$$\beta_{\gamma'} = -\frac{1}{\Lambda_1} + \beta_{\gamma''}, \quad (10)$$

and

$$\beta_{\gamma''} = -\frac{4E_p}{3}\frac{\mu_{68}}{m_0}\left(\frac{8}{(E_g^0)^2} + \frac{1}{(E_g^{0\prime})^2} + \frac{2}{E_g^0E_g^{0\prime}}\right). \quad (11)$$

The renormalized fundamental gap may be obtained from equation (6) as

$$E_g = E_+(0) - E_-(0)$$
$$= E_g^0 - \delta + \sqrt{\frac{8\Lambda_0^2}{3} + \left(\delta + \frac{\Lambda_0^2}{3\delta'} + \frac{4\Lambda_0^2}{3\Lambda_1}\right)^2}, \quad (12)$$

where Λ_0 is related to the laser intensity [11], i.e.

$$\Lambda_0 = \left[\left(\frac{2I}{I_0}\right)\frac{p^2}{m_0}E_g^0\right]^{1/2}, \quad (13)$$

with $I_0 \approx 5 \times 10^{13}$ W cm^{-2}.

2.2. The impurity problem

Here we have considered a GaAs–$Ga_{1-x}Al_xAs$ cylindrical QWW with radius R under the influence of an external magnetic field applied along the wire axis, and a hydrogenic shallow donor located at the position $\vec{r}_0$ in the wire. If one also considers the effects of a homogeneous laser field acting upon the system, the Hamiltonian describing the impurity states in the QWW is given by

$$\hat{H} = \frac{\hat{\vec{P}}^2}{2m^*} - \frac{e^2}{\epsilon|\vec{r} - \vec{r}_0|} + V(\vec{r}), \quad (14)$$

where $\hat{\vec{P}} = \hat{\vec{p}} + \frac{e}{c}\vec{A}(\mathbf{r})$, $\vec{A}(\vec{r}) = \frac{1}{2}\vec{B} \times \vec{r}$ is the vector potential, ϵ is the static dielectric constant, $m^* = m_+$ (cf equation (7)) is the renormalized conduction-band effective mass due to the laser-field effects [12] (both the static dielectric constant and the electron conduction-band effective mass are assumed, for simplicity, to be constants throughout the heterostructure), and V is the renormalized confining potential [11–13] corresponding to the GaAs–$Ga_{1-x}Al_xAs$ cylindrical QWW, which is zero in the region $\rho < R$ and V_0 if $\rho > R$. In the present study, we have considered the impurity located over the wire axis, at the origin of the coordinates, so $\vec{r}_0 = 0$.

The characteristic problem for the Hamiltonian (14) may be solved by proposing a suitable envelope wavefunction associated with the impurity states in the semiconductor QWW. Following the procedure developed by Spiros *et al* [5], the wavefunction for a shallow impurity associated with the lowest electron level in the QWW may be chosen as the product of a hydrogenic part, involving the effects of the Coulomb interaction, and a part which is the ground-state solution for an electron moving in a cylindrical QWW under the influence of a magnetic field applied parallel to the wire axis in the absence of the Coulomb potential, i.e.

$$\Phi(\vec{r}) = \Psi(\rho)f(\vec{r}), \quad (15)$$

where

$$\Psi(\rho) = N\exp\left[-\frac{\rho^2}{4l_B^2}\right] \times \begin{cases} {}_1F_1\left(-\alpha, 1, \frac{\rho^2}{2l_B^2}\right) & \text{if } 0 < \rho < R \\ \frac{{}_1F_1(-\alpha, 1, \xi_R)}{U(-\alpha', 1, \xi_R)}U\left(-\alpha', 1, \frac{\rho^2}{2l_B^2}\right) & \text{if } \rho > R, \end{cases} \quad (16)$$

N is a normalization constant, $\rho = \sqrt{x^2 + y^2}$, l_B is the Landau length, ${}_1F_1$ and U are the hypergeometric confluent functions of first and second kinds [16], respectively, and $\xi_R = \frac{R^2}{2l_B^2}$. The coefficients α and α' satisfy the expression $\alpha - \alpha' = \frac{V_0}{\hbar\omega_c}$, where ω_c is the cyclotron frequency, and α may be computed from the condition of the continuity of the probability current at the interface $\rho = R$, i.e. $\frac{d\Psi}{d\rho}|_{\rho=R^-} = \frac{d\Psi}{d\rho}|_{\rho=R^+}$. In that way, one obtains α as the first zero of the transcendental equation

$$\left(\alpha - \frac{V_0}{\hbar\omega_c}\right){}_1F_1(-\alpha, 1, \xi_R)U\left(-\alpha + 1 + \frac{V_0}{\hbar\omega_c}, 2, \xi_R\right)$$
$$+ \alpha\,{}_1F_1(-\alpha + 1, 2, \xi_R)U\left(-\alpha + \frac{V_0}{\hbar\omega_c}, 1, \xi_R\right) = 0. \quad (17)$$

By substituting the wavefunction (15) into the characteristic problem for the Hamiltonian (14), one may straightforwardly obtain that the wavefunctions f satisfy the Schrödinger equation

$$\hat{\mathcal{H}}f(\vec{r}) = Ef(\vec{r}), \quad (18)$$

where

$$\hat{\mathcal{H}} = -\frac{\hbar^2}{2m^*}\frac{1}{h(\rho)}\nabla \cdot h(\rho)\nabla + \frac{\omega_c}{2}\hat{L}_z - \frac{e^2}{\epsilon\sqrt{\rho^2 + z^2}}, \quad (19)$$

$h(\rho) = \Psi^2(\rho)$, $\hat{L}_z$ is the z-part of the angular momentum operator, and E is the impurity energy measured with respect to the ground-state electron energy $E_0 = \hbar\omega_c(\alpha + 1/2)$ in the QWW under a magnetic field applied along the wire axis and in the absence of the Coulomb potential. One may note that equation (18) is an exact equation, and the wavefunctions f are orthogonal with weight h. However, as the solution of the differential equation (18) is unknown, we have used the variational scheme in order to obtain the energies of the 1s ground state and the $2p_\pm$ states corresponding to the shallow donor in the QWW. We choose the single-parameter trial wavefunctions, written in cylindrical coordinates, as $f_{1s}(\rho, \varphi, z) = A_{1s}\exp(-\lambda_{1s}\sqrt{\rho^2+z^2})$ and $f_{2p_\pm}(\rho, \varphi, z) = A_{2p_\pm}\rho\exp(-\lambda_{2p_\pm}\sqrt{\rho^2+z^2} \pm i\varphi)$ for the 1s and $2p_\pm$ impurity-electron states, respectively. The appropriate value of the variational parameter λ_l^0, where $l = 1s$, $l = 2p_+$, or $l = 2p_-$, may be obtained from the minimization of the functional

$$\mathcal{E}(\lambda_l) = \int h(\rho) f_l^*(\vec{\mathbf{r}})\hat{\mathcal{H}} f_l(\vec{\mathbf{r}})\,d^3\vec{\mathbf{r}}, \tag{20}$$

and the impurity binding energy corresponding to the 1s, $2p_+$, or $2p_-$ impurity states may then be obtained as

$$E_B(l) = -\mathcal{E}(\lambda_l^0). \tag{21}$$

3. Results and discussion

We have first calculated the ground-state binding energy of an on-center shallow impurity, as a function of the magnetic field applied along the wire axis, in GaAs–$Ga_{0.7}Al_{0.3}As$ QWWs, for two different values of the wire radius. Numerical results are displayed in figure 1, in which solid, dashed, and dotted lines were obtained by considering the presence of a homogeneous laser field with intensities $I = 0$, $I = 0.5 \times 10^{-4} I_0$, and $I = 10^{-4} I_0$, respectively, acting upon the system, and a laser detuning $\delta = 0.01 E_g^0$. One may note that, for the two wire radius analyzed here, only the 1s electron state is a bound state in the full range of the magnetic field considered in the present study, i.e. it is an impurity state with a positive binding energy. On the other hand, the $2p_\pm$ states remain unbound (negative binding energy) in the same range of applied magnetic fields studied here. In the case where the wire radius is comparable with the impurity Bohr radius ($a_B = \hbar^2\epsilon/m_{\Gamma_6^c}e^2$), the $2p_-$ state may be a bound state beyond a certain value of the applied magnetic field which depends on the wire radius (results are not shown here). A similar result was obtained by Villamil and Porras-Montenegro [17] for a shallow donor in GaAs–$Ga_{1-x}Al_xAs$ QWWs with $R \simeq a_B$ in the absence of laser radiation. Moreover, the ground-state donor binding energy is a growing function of the magnetic field both in the absence and presence of laser radiation, a well-known result which is related to the increasing in the ground-state confinement as the magnetic field is increased, and also found [15] in single QWs. The $2p_-$ binding energy is a growing function of the applied magnetic field, whereas the $2p_+$ binding energy decreases monotonically as a function of the magnetic-field strength. The above behavior of the donor binding energy may be understood in terms of the electron-localization properties in the QWW, which are firstly modified by changes in the conduction-band effective mass and confining potential due to the laser radiation, and secondly by the magnetic field applied along the wire axis. It is known that the effective mass increases (decreases) as the laser intensity (detuning) is increased, whereas the potential height behaves in an opposite way. An increase (decrease) of the conduction-band effective mass or potential-barrier height leads to an increase (decrease) of the impurity-electron localization and, therefore, to an increase (decrease) of the donor binding energy. In addition,

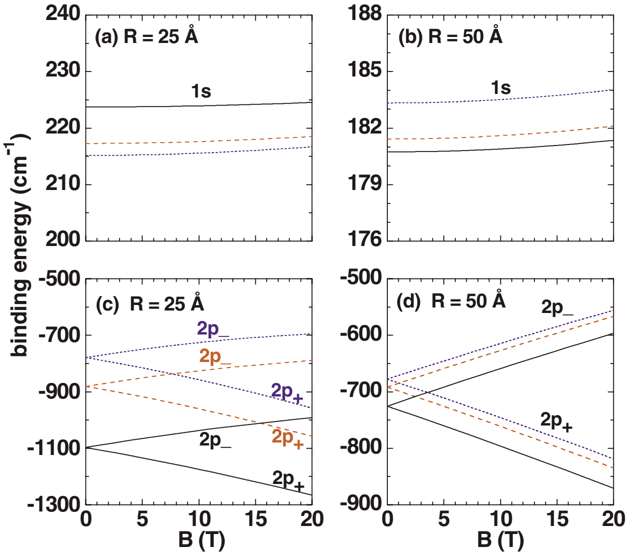

Figure 1. Magnetic-field effects on the on-center donor binding energy, associated with the 1s and $2p_\pm$ states in semiconductor GaAs–$Ga_{0.7}Al_{0.3}As$ QWWs for two different values of the wire radius. Calculations were performed for $\delta = 0.01E_g^0$, where E_g^0 is the GaAs gap in the absence of the laser radiation. Solid, dashed, and dotted lines correspond to laser intensities $I = 0$, $I = 0.5 \times 10^{-4} I_0$, and $I = 10^{-4} I_0$, respectively, where I_0 is a characteristic value of the laser intensity (see the text).

an increase of the magnetic field tends to localize the donor–electron wavefunction around the impurity nucleus and to an increase of the donor binding energy. Thus, when laser radiation and magnetic field are simultaneously acting upon the 'impurity + QWW' system, the impurity energies result from the competition between the above-mentioned effects.

One may see, for $R = 50$ Å, that laser-field effects result in an increase of the 1s and $2p_\pm$ shallow-donor binding energies with increasing laser intensities (cf figures 1(b) and (d)). A similar situation is also observed for the $R = 25$ Å $2p_\pm$ binding energies displayed in figure 1(c), whereas the $R = 25$ Å 1s-state binding energy decreases with increasing laser intensities, as one may note from figure 1(a). This behavior may be understood from figure 2, where we have displayed the 1s on-center donor binding energy in GaAs–$Ga_{0.7}Al_{0.3}As$ QWWs as a function of the wire radius. The applied magnetic field and laser detuning were taken as $B = 4$ T and $\delta = 0.01E_g^0$, respectively. Solid, dashed, and dotted lines correspond to laser intensities $I = 0$, $I = 0.5 \times 10^{-4} I_0$, and $I = 10^{-4} I_0$, respectively. In the absence of laser radiation and for the lowest values of the wire radius, the donor–electron wavefunction may easily tunnel through the potential barrier, it is spread into the $Ga_{0.7}Al_{0.3}As$ material, and localized in a spatial region of the order of the $Ga_{0.7}Al_{0.3}As$ impurity Bohr radius around the positive nucleus. As the wire radius is increased, the transmission probability through the barrier diminishes, the electron wavefunction becomes more localized, and the binding energy increases until a certain value of the wire radius for which the influence of the barrier potential on the wavefunction becomes less important. Starting from this point, as the wire radius increases, the donor–electron

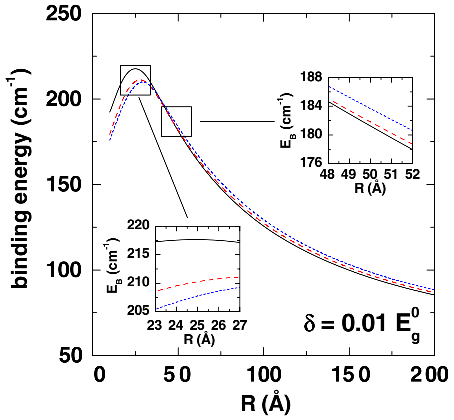

Figure 2. 1s donor binging energy, as a function of the wire radius, associated with a shallow donor located at the center of GaAs–$Ga_{0.7}Al_{0.3}$As QWWs. Solid, dashed, and dotted lines correspond to laser intensities $I = 0$, $I = 0.5 \times 10^{-4} I_0$, and $I = 10^{-4} I_0$, respectively. Calculations were performed for $\delta = 0.01 E_g^0$ and $B = 4$ T.

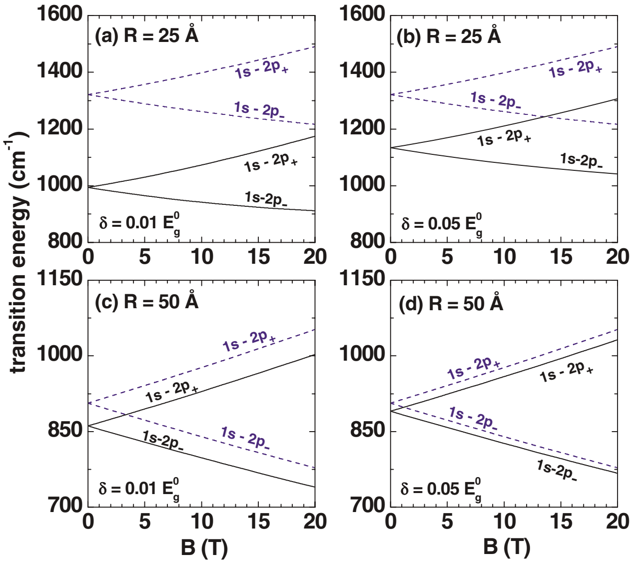

Figure 3. $1s - 2p_-$ and $1s - 2p_+$ transition energies, as functions of the applied magnetic field, corresponding to a shallow donor located at the center of GaAs–$Ga_{0.7}Al_{0.3}$As QWWs, for two different values of the wire radius. Calculations were performed for $\delta = 0.01 E_g^0$ and $\delta = 0.05 E_g^0$, and for laser intensities $I = 0$ (dashed lines) and $I = 10^{-4} I_0$ (solid lines).

wavefunction begins to localize around the impurity nucleus in a spatial region of the order of the GaAs effective Bohr radius, and the binding energy diminishes and tends toward the GaAs effective Rydberg. In GaAs-$Ga_{0.7}Al_{0.3}$As QWWs the position of the binding energy maximum is at a QWW radius $R \sim 25$ Å in the absence of laser radiation. When a laser field is acting upon the system, the behavior of the donor binding energy as a function of the wire radius is essentially the same but, as the laser radiation modifies both the electron effective mass and confining potential, the maximum of the binding energy is shifted to the right for given values of the laser intensity and detuning. This is the reason why laser-field effects, for increasing laser intensities, lead to a decreasing of the 1s binding energy for $R = 25$ Å and to an increasing of the binding energy for $R = 50$ Å, as one may note from the insets in figure 2.

The magnetic-field dependence of the transition energy between the 1s and $2p_\pm$ impurity states in GaAs–$Ga_{0.7}Al_{0.3}$As QWWs is displayed in figure 3 for two different values of the wire radius. Dashed and solid lines correspond to numerical results obtained for the laser intensities $I = 0$ and $I = 10^{-4} I_0$, respectively, and for two values of the laser detuning, $\delta = 0.01 E_g^0$ and $\delta = 0.05 E_g^0$. One may note from figure 3 that the presence of the laser radiation leads to a decreasing of the 1s–$2p_\pm$ transition energies for a given value of the applied magnetic field, a phenomenon which is more remarkable for the lowest values of the wire radius and laser detuning, i.e. the applied laser induces a red-shift in the 1s–$2p_\pm$ transition energies which depends on the laser intensity, detuning, and wire radius. It is apparent from figure 3 that, for a given value of the laser intensity, the shift of the transition energies is less important as the laser detuning or wire radius increase.

The dependence of the 1s–$2p_-$ and 1s–$2p_+$ transition energies on the laser intensity and detuning is shown in figure 4 for three different values of the wire radius. Calculations were performed for an on-center shallow donor in GaAs–$Ga_{0.7}Al_{0.3}$As QWWs at $B = 4$ T. One may note from figure 4(a) that the 1s–$2p_\pm$ transition energies decrease as the laser intensity is increased, as mentioned. However, the negative slopes associated with the curves displayed in figure 4(a) increase as the wire radius is increased until they reach positive values at $R = 100$ Å (cf figure 4(e)). In this case both the $1s - 2p_-$ and $1s - 2p_+$ transition energies increase as the laser intensity is increased. The dependence of the 1s–$2p_\pm$ transition energies on the laser intensity is determined, therefore, by the value of the wire radius. For the lowest values of the wire radius the laser radiation induces a red-shift in the 1s–$2p_\pm$ transition energies, whereas for sufficiently large values of the wire radius ($R \simeq a_B$) a blue-shift in the 1s–$2p_\pm$ transition energies may take place. The 1s–$2p_\pm$ transition energies also increase as the laser detuning is increased, but they reach a saturation value beyond a certain critical laser detuning (cf figures 4(b), (d), and (f)). It is apparent from figure 4 that both the critical value of the detuning for which the saturation occurs and the saturation values of the transition energies are functions of the wire radius. One may note that the δ dependence on the transition energies is more remarkable for the lowest values of the wire radius, and it is almost flat for larger values of R. The behavior of the 1s–$2p_\pm$ transition energies as a function of the laser detuning is essentially due to the saturation observed both in the electron effective mass and confining potential height as functions of the laser detuning, as we have discussed in a previous work [18]. The resulting values of the renormalized effective mass and barrier height, together with the value of the wire radius and applied magnetic field, determine the electron-localization properties in the QWW and its corresponding transition energies, as displayed in figure 4.

It is important to point out that, in the case of QWs, laser-field effects on the impurity states must be taken into account in

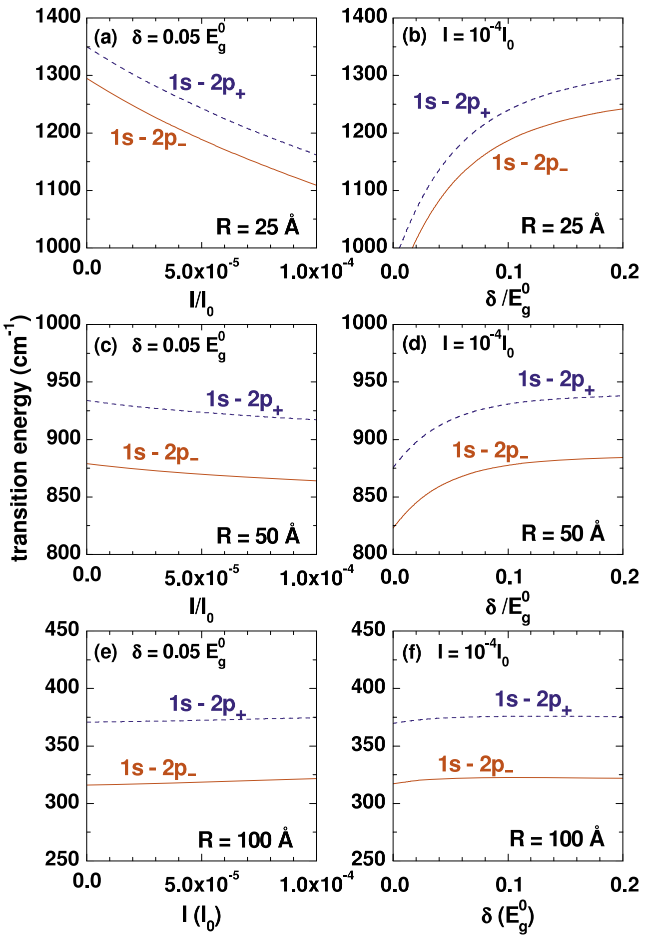

Figure 4. On-center shallow-donor $1s - 2p_-$ and $1s - 2p_+$ transition energies, as functions of the laser intensity, for a fixed value of the laser detuning $\delta = 0.05E_g^0$ ((a), (c), and (e)), and as functions of the detuning, for a fixed value of the laser intensity $I = 10^{-4}I_0$ ((b), (d), and (f)), in GaAs–$Ga_{0.7}Al_{0.3}As$ QWWs. Calculations were performed for three different values of the wire radius and for $B = 4$ T.

order to obtain quantitative agreement between theoretical and experimental results [11–13]. Although in the present work we have not carried out any comparison between theoretical calculations and experimental measurements (we do not know of any experimental studies of laser effects in doped QWWs), we expect that, as in QW heterostructures, the effects of the laser radiation on the donor states in QWWs would be important for the understanding of the impurity properties in such systems.

4. Conclusions

Summing up, we have presented a theoretical study of the laser-field effects on shallow-donor states in semiconductor GaAs–$Ga_{1-x}Al_xAs$ QWWs under magnetic fields applied along the wire axis. The present theoretical results were obtained within the extended dressed-atom approach, a procedure which allows one to replace the actual quantum-mechanical problem, involving the shallow impurity in a semiconductor heterostructure under a laser field and an applied magnetic field, by the simplest problem involving only a shallow impurity in the semiconductor heterostructure under a magnetic field, with laser-field effects incorporated via a renormalization of both the conduction-band effective mass and the fundamental semiconductor gap. We have used the variational scheme to obtain the ground-state binding energy as well as the $2p_\pm$ binding energies associated with a shallow donor located at the center of the wire. A shift of the binding energy is observed when the laser field is applied, which may be understood in terms of the electron-localization properties of the QWW. We have also shown that the laser intensity dependence of the $1s–2p_\pm$ transition energies is determined by the value of the wire radius. For a fixed value of the laser detuning and for the lowest values of the wire radius the transition energies decrease monotonically as the laser intensity is increased. Such a dependence is reversed for sufficiently large values of the wire radius, i.e. for $R \simeq a_B$, where a_B is the impurity Bohr radius in bulk GaAs without the laser influence. One may therefore obtain either a red- or blue-shift in the $1s–2p_\pm$ transition energies by appropriately changing the value of the wire radius. Finally we have discussed the saturation taking place in the $1s–2p_\pm$ transition energies as functions of the laser detuning for given values of the laser intensity and applied magnetic field. We do hope that the knowledge of laser-field effects on the shallow-impurity states provides a useful tool to obtain a quantitative understanding of the experimental measurements of impurity properties in semiconductor QWWs.

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